COMPLETE DEVELOPMENT OF AN BATTERY CHARGER SYSTEM
WITH STATE-OF-CHARGE ANALYSIS

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Abstract
Battery charging and State-Of-Charge management is one of the main relevant issues for developing mobile autonomous robots. This paper presents a complete development of an battery charger for an autonomous mobile robot and a discussion of SOC estimation models. Initially a charger prototype, based on a Half-Bridge DC-DC converter, was developed and tested. Furthermore, the MATLAB SimPowerSystems default battery model was analyzed and a limitation of the model concerning the effect of battery internal resistance is then discussed. Afterwards, a bench test was made by measuring charge voltage and current in a prototype board. After testing was through, it was observed that the battery model used in SimPowerSystems is not accurate due to the assumption of a constant battery internal resistance. Then, a variable resistance modification to this model was introduced and a new bench test showed satisfactory results. The variable resistance model provides a correction for both charge and discharge models for a lead acid battery, thus improving the accuracy of the autonomous robot autonomy, as well as avoiding excessive battery depth of charge, which is a limiting factor for their economic life and serviceability. Whereas such results are of limited reach in an applied research project, they mean economic scale gains for the widespread industrial lead-acid battery applications, hence a better battery model, shall allow more accurate battery serviceability, on time battery inspections and maintenance of their elements, and thus a lesser rate of disposal of lead sulphates and plastic casings into the environment, when disposing used lead-acid batteries.

Keywords: – Battery charger, state-of-charge estimation.

I. INTRODUCTION
With increasing emissions of the greenhouse gases and the global warming, plus the concern combined with the depletion of fossil fuels, many countries have started to develop hybrid/electric cars. This new type of automobiles relies on a complex battery system to power the main engine. Therefore, an intelligent battery charging is an important issue in the design of modern electro-electronics systems.

The battery state-of-charge (SOC) is a critical parameter to inform the driver when he needs to recharge the vehicle’s battery system for both practical and economic reasons. Most of the commercially available battery chargers are based on a simple architecture composed by an insulated transformer and a bridge
rectifier. While it is cheap, this type of battery charger cannot perform well by design and it can also contribute to a lesser serviceability of the automobile’s battery due to lack of charge current control. Also, there are many types of intelligent battery chargers available in the market, which cannot recharge a deeply discharged battery.

In principle, any battery charger controller has to estimate the battery SOC in order to regulate the charge current, and such problem has been widely discussed in the scientific community in the past years. The main relevant techniques used to estimate the state-of-charge of a battery were: Application of Extended Kalman Filter (EKF) [1], Particle Filter (PF) [2], Coulomb Counting by integrating the charging current, Fuzzy Logic Inference System [3], H-infinity Filter [4].

In the MATLAB software the Simulink tool has a battery model based on ref. [7]. This model has an important drawback modeling the battery internal resistance as a constant, which is not true for the lead-acid battery tested in this paper.

An extension of this model, using an adaptive resistance model based on the EKF, will be proposed in the next sessions of this paper along with the results. It is assumed in this paper that, whereas the EKP can estimate the continuous value of the resistance, a best fitting polynomial, piecewise continuous model, shall be adopted for this study.

The purpose of the latter model is to simplify the programming efforts plus the size, complexity and the costs of the FPGA in which to implement the controller.

It must be forewarned here that the lead acid batteries used in this study are rated 7Ah 12V, such that the much higher rating 28V batteries of the automotive or industrial use shall naturally present different coefficients. These same considerations shall also apply to the hydrogen technology batteries of the automotive industry, as well.

II. BATTERY CHARGER

A Half-Bridge DC-DC Converter was developed to convert the 380VDC, rectified from the mains AC, to 0~50VDC. The battery charger was developed in four fundamental modules described by Figure 1:

![Battery Charger Block Diagram](image)

Fig. 1. Battery Charger Block Diagram

After the conceptual design phase, a prototype was built. This experimental prototype is shown in the Figure 2.
The Half-Bridge converter was tested and it was found that the circuit works as expected. In the first tests it was used an SG3524 integrated circuit to generate the PWM for the MOSFETs. This converter generated over 40VDC@9.5A when the PWM duty cycle was set to 45%.

In the PSIM environment of MATLAB Simulink, then the Half-Bridge converter was simulated in an open loop configuration. The software start system of the Half-Bridge converter was implemented to slow down the converter’s startup in order to minimize the wear down in the electronic components due to inrush current. Fig. 3 shows this simulation, including the soft-start system.

By simulating the battery charger in an open loop configuration, another simulation was done in closed loop by also adding a WGN generator at the output to evaluate the behavior of the system controller.
In Fig. 4 simulation, a proportional-integral (PI) controller was implemented to improve the plant stability and to maintain the output voltage in a steady level. Note that the pulse duty cycle was set to its maximum to analyze the maximum battery charge performance.

The lead acid batteries used in cars and wheelchairs represent a very slow case of a control plant. Normally, it is not necessary to use a very high sampling rate in such control systems due to this aspect.

For lead acid batteries, when discharged to 20% of their rated capacity. The recharge current will be generally very high, even for these low capacities 12V, 7Ah batteries. This phenomenon occurs due to the fact the internal resistance of the battery under a low charge state decrease. Because of this aspect, we have designed a current limiting circuit load to limit the recharge current to 5A, even if a short circuit should pop up on the output terminals of the DC-DC converter. Fig. 5 shows this circuit, as series implemented in the output of a DC-DC converter to the battery terminals.

After testing this circuit in a prototype PCB board it was demonstrated that the short circuit current passing through the terminal BATT[+] was stabilized in over 4.86A. This measure was done using a DC variable power supply and a shunt resistor of 750mohm connected directly to the BATT [+] and BATT [-] terminals. After measuring the voltage between the shunt resistor terminals, it was noted that the registered voltage were over 3.645V.
Once done the MATLAB Simulink simulation of DC-DC battery charger, the battery charger controller was tested to try to charge four old deeply discharged batteries.

This test was made to verify the aspect of the high recharge currents and the effect of the wear down of the batteries. These are the type VRLA batteries from 7Ah@12V. Table I presents the voltages and relevant data of these batteries.

**TABLE I**
**Battery Data**

<table>
<thead>
<tr>
<th>List of Batteries</th>
<th>Battery ID</th>
<th>Manufacturer</th>
<th>Voltage</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Alpha-1</td>
<td>RONTEK</td>
<td>4.5</td>
<td>Deeply Discharged 0%</td>
</tr>
<tr>
<td></td>
<td>Alpha-2</td>
<td>UNIPOWER</td>
<td>4.2</td>
<td>Deeply Discharged 0%</td>
</tr>
<tr>
<td></td>
<td>Alpha-3</td>
<td>UNIPOWER</td>
<td>2.8</td>
<td>Deeply Discharged 0%</td>
</tr>
<tr>
<td></td>
<td>Alpha-4</td>
<td>UNIPOWER</td>
<td>2.8</td>
<td>Deeply Discharged 0%</td>
</tr>
</tbody>
</table>

The lead acid batteries normally require a constant charging voltage equal to 13.4~13.8V. Therefore, the battery charger controller was designed to deliver to the batteries a voltage in this range. This is achieved by lowering the pulse width.

Before testing these batteries with the prototype battery charger, it was used a variable DC power supply to perform the initial charge. This test was chosen to evaluate the controller’s performance. Further, the ATMega328 kit was used combined with a shunt resistor of 75mV/A.

The ATMega328 kit had an integrated SD card to store the charge current over the time at a sample rate of 1 kHz. The batteries, of Table I, were in a deeply discharged state and after the charge procedure on each battery, it was noted that the charging current remained constant and equal to approximately 60mA. This initial test failed to obtain the batteries charge curve.

The impedance of each individual battery was so high that it can be stated that this charger cannot recharge batteries that have been deeply discharged, once that the chemical reactions within them have become irreversible. This also, confirm a limitation of the [7] battery model used on Sim Power Systems.

**IV. STATE-OF-CHARGE ANALYSIS**

The effectiveness of the SOC algorithms will be discussed in this section. Previously, an attempt was made to obtain the charge/discharge curve of a sample of four fully discharged lead acid batteries. This test,
as described above, did not achieve satisfactory results due to the very high internal impedance of these batteries. This fact highlights the important issue of not allowing any battery to be deeply discharged.

Many authors have discussed the issue of creating a precise model that can better represent the battery discharge/charge behavior. One of the most common techniques used to measure the battery SOC is to count how many Coulombs have entered or exited the battery [11]. Eq. 1 shows how to perform the Coulomb counting.

\[ q(t) = \int_{0}^{t} i(t) dt \]  

According to [2] this model, presented by Eq. 1, can be generalized and discretized to produce a function which represents the SOC over the time. Then, Eq. 2 and Eq. 3 show the function \( z(t) \), representing the SOC over the time.

\[ z(t) = z(0) - \frac{\eta(t) \Delta t}{C} \]  

\[ z_{k+1} = z_k - \frac{\eta \Delta i}{C} i_k \] 

Ref. [7] presents a generic battery discharge/charge model that can be used to describe the lead-acid battery used in this work. Eq. 4 and Eq. 5 show the battery voltage equation of a generic battery discharge and charge, respectively.

\[ V_{\text{batt}} = E_0 - R i - K \frac{Q}{Q - it} \cdot (it + i^*) + \text{Exp}(t) \]  

\[ V_{\text{batt}} = E_0 - R i - K \frac{Q}{it - 0.1Q} i^* - K \frac{Q}{Q - it} it + \text{Exp}(t) \] 

Where:
- \( V_{\text{batt}} \) = Battery Voltage (V)
- \( E_0 \) = Battery Constant Voltage (V)
- \( K \) = Battery Polarization Constant (V/Ah)
- \( Q \) = Battery Nominal Capacity (Ah)
- \( it = \int_{0}^{t} idt \) = Actual Battery Charge (Ah)
- \( R \) = Internal Resistance (Ω)
- \( i \) = Battery current (A)
- \( i^* \) = Battery Filtered Current (A)

However, this model, previously presented with Eq. 1, has an important drawback. The actual lead-acid batteries, which are used in this work, exhibit a hysteresis in the charge and discharge curves in all SOC conditions [7-9].
Then, this phenomenon can be represented by the Eq. 6 where the exponential voltage can be calculated more precisely in this case [7].

\[ \text{Exp}(t)_{\text{leadacid}} = B \times i(t) \times (-\text{Exp}(t) + Au(t)) \quad (6) \]

Where:
\( \text{Exp}(t) \) = Exponential zone voltage (V)
\( i(t) \) = battery current (A)
\( u(t) \) = charge or discharge mode (\( u(t)=1 \) charging; \( u(t)=0 \) discharging)
\( A \) = Batt. Voltage exponential zone amplitude (V)
\( B \) = Batt. Voltage exponential zone time constant inverse (Ah\(^{-1}\))

We can observe in this model that the battery internal resistance is assumed as Time-Invariant. In a real battery, the internal resistance is known to vary greatly by a number of factors. Those factors include: Decrease of the useful life (wear down of battery), Temperature changes, charge/discharge current and also the State-Of-Charge [10].

The very first experiment in this work, showed the typical wear down phenomena of the battery: - all four tested batteries had a large internal resistance, which preventing further charging.

After that, within the MATLAB computational environment, a model was built, using Ref [7], for charging and discharging of a brand new lead battery acid 12V@1.3Ah.

Initially, we simulated a fixed load connected to the battery’s terminals, draining 1A during 1 hour. Then, in the laboratory, the actual battery was discharged using a fixed load of 1A. Figs. 6, 7, and 8 show the battery discharge curve, current drained over the time and the state-of-charge (SOC). The graphs illustrate the simulated vs. real laboratory test.

![Battery discharge curve under fixed draining 1A](image-url)

Fig. 6. Battery discharge curve under fixed draining 1A
In the MATLAB simulation we note that the internal resistance of the battery is constant, since the drained current and voltage from the battery have the same characteristic shape, however, as shown by the lab results, is untrue in a real lead-acid battery.

Some authors have then developed models that include the battery internal resistance. Ref. [14] shows a model that describes the battery internal resistance considering the state of charge and other parameters. This model, that considers the internal resistance and the terminal resistance by using three components. According to Ref. [14] Eq. 7, 8 and 9 describe the battery terminal resistance ($R_t$) and battery internal resistance considering the SOC.

$$R_t = R_{ti}[1 + \alpha(1 - SOC)] \quad (7)$$

$$R_u = -R_{ui} \ln(DOC) \quad (8)$$

$$R_b = R_{bi}\frac{\exp[\beta(1 - SOC)]}{1 + \exp(\lambda I_m / T)} \quad (9)$$

Where:

$R_t$ = Terminal Resistance (\Omega)

$R_{ti}$ = Initial Terminal Resistance (\Omega)
\[ R_a = \text{Internal Resistance Component A} \ (\Omega) \]
\[ R_b = \text{Internal Resistance Component B} \ (\Omega) \]
\[ R_{ai} = \text{Internal Initial Resistance Component A} \ (\Omega) \]
\[ R_{bi} = \text{Internal Initial Resistance Component B} \ (\Omega) \]
\[ \alpha = \beta = \gamma = \text{System Constants (Dimensionless)} \]
\[ DOC = \text{Depth of Charge (Dimensionless)} \]

Then, the lead-acid battery model proposed by Ref. [7] could be modified by considering the combination of the three resistive components described by Eqs. 7, 8 and 9. The \( R \) parameter of Eqs. 4 and 5 will be assumed as this combination. These resistors, as modeled by Ref. [14], are in a series-parallel configuration. Fig. 9 shows the circuit, developed by Ref. [14] describing the lead-acid battery equivalent circuit.

![Circuit Diagram](image)

Fig. 9. SOC of the battery over the time

This circuit of Fig. 9 was simulated using the PSIM environment of MATLAB. The \((RA)\) resistor was modeled to be non-linear in order to reach a battery voltage curve similar to Fig 6. The value for capacitor \(C1\) was 2000F, \(RB\) was chosen as 1m\(\Omega\), \(TP\) was set at 10k\(\Omega\), \(C2\) was set as 10pF and \(RC\) was set at 1m\(\Omega\).

After entering these values to the test battery set-up, the results achieved are presented in Simulation B by Fig. 10.

![Battery Voltage Graph](image)

Fig. 10. SOC of the battery over the time

In Fig. 10 it is seen that the battery model presented by Ref [14] has a fairly accurate result compared to the model developed by Ref [7]. However, both such models show the critical drawback of the battery’s constant internal impedance in function of the available SOC.
Thus, taking this effect of the non-constant battery’s internal impedance, the model presented by Ref [14] was modified by including the effect of a variable parameters for RA, C1, RB, TP, C2 and RC, and then estimating through the EKF their values, based on the actual discharge curve of the 1.3Ah, 12V battery.

Then, the new voltage curve is presented by “Sim. C” of the Fig. 11.

This new voltage curve of Simulation C is slightly more accurate than the “ideal” curves, presented by the previously made Simulation A and B. The Simulation C does not consider the effect of the temperature of the battery lead-acid solution. This parameter also contributes greatly to the increase of the battery’s internal impedance.

Then, after collecting the samples of the voltage and current from the battery, used in the analysis, the internal resistance was estimated from the voltage and current data. As we can see, the lead-acid battery model developed by [7] will always consider the internal resistance constant. In the measurements, it is possible to see that the battery internal resistance drops over the time and when the SOC reaches over 50% the internal resistance will increase drastically.

The Fig 12 illustrates the effect of the battery internal resistance variation under the fixed load of 1A drained during one hour.

In the graph of Fig 12 the internal resistance will normally drop during the first 30 minutes of discharge. When the battery operates next to 50% of SOC the internal resistance of the battery will increase reducing
the capacity of delivering current. Thus, the nominal voltage, in this condition, will drop drastically as presented by the Fig. 10.

This effect of the internal resistance increase can be explained due to the influence of the temperature of the battery acid solution and the terminal temperature [6]. Also, the Peukert effect [10] has a small role here even if the discharge current is almost fixed to 1A.

V. IDENTIFICATION OF THE BATTERY DISCHARGE MODEL

Once the ad hoc strategy of Section IV has not yielded a straightforward solution, it shall be presented here another strategy based on a variation of the Least Square fit applied to the Battery discharge model of Ref [7].

This strategy can be easily adapted for the Battery charge model, but this subject shall be discussed in another paper.

Then, let’s state the following problem based on Ref [7]: Minimize,

\[ I_p = \sum \left( V_{\text{batt}} - E_0 - RI_i - \frac{KQ(I_t)}{Q-I_t} + B \left| I_i \right| \times \exp(t_i) \right)^2 \]  \hspace{0.5cm} (10)

Where R, the battery internal resistance, is dynamically modeled in this paper as \( R \equiv a_0 + a_1 V_{\text{bat}, i} + 0.50 a_2 V_{\text{bat}, i}^2 \), by author’s judgment after a look-up at Figure 6 and Figure 7. The physical reasons behind such behavior are not taken into account here as in Ref [14].

Therefore, it remains to solve the problem such that the optimum set of Ref [7] model parameters \( \{a_0, a_1, a_2, K, B\} \) is determined such that the 4 segments of the curve Vbat x time of Figure 11 are fully tracked by the model from end to end, and not just partially, as seen per the ad hoc approach of Section IV.

The necessary conditions for the existence of the minimum and optimum set of model parameters \( \{a_0, a_1, a_2, K, B\}^* \) are given by,

\[ \frac{\partial I_p}{\partial a_0} = 0; \frac{\partial I_p}{\partial a_1} = 0; \frac{\partial I_p}{\partial a_2} = 0; \frac{\partial I_p}{\partial K} = 0; \frac{\partial I_p}{\partial B} = 0 \] \hspace{0.5cm} (11)

This can, then, be individually expressed as,

\[ \frac{\partial I_p}{\partial a_0} = \sum \{ V_{\text{batt}} - E_0 - RI_i - KQ(I_t + I)/[Q-I_t] + B \left| I_i \right| \times \exp(t_i) \times 1 \} = 0 \]

\[ \frac{\partial I_p}{\partial a_1} = \sum \{ V_{\text{batt}} - E_0 - RI_i - KQ(I_t + I)/[Q-I_t] + B \left| I_i \right| \times \exp(t_i) \times V_{\text{batt}} \} = 0 \]

\[ \frac{\partial I_p}{\partial a_2} = \sum \{ V_{\text{batt}} - E_0 - RI_i - KQ(I_t + I)/[Q-I_t] + B \left| I_i \right| \times \exp(t_i) \times 0.5V_{\text{batt}}^2 \} = 0 \]

\[ \frac{\partial I_p}{\partial K} = \sum \{ V_{\text{batt}} - E_0 - RI_i - KQ(I_t + I)/[Q-I_t] + B \left| I_i \right| \times \exp(t_i) \} \times Q(I_t + I)/[Q-I_t] = 0 \]

\[ \frac{\partial I_p}{\partial B} = \sum \{ V_{\text{batt}} - E_0 - RI_i - KQ(I_t + I)/[Q-I_t] + B \left| I_i \right| \times \exp(t_i) \times I_i \} \times \exp(t_i) = 0 \]
This whole set of equations can then be organized into a linear matrix format such that $Ax = b$, where the $A$ matrix is,

$$
A = \begin{bmatrix}
\sum_{i=1}^{N} V_{bat,i} & \sum_{i=1}^{N} 0.5V_{bat,i} & \sum_{i=1}^{N} Q_{i} + I_{i} & \sum_{i=1}^{N} abs(I_{i})exp(t_{i}) \\
\sum_{i=1}^{N} V_{bat,i}^2 & \sum_{i=1}^{N} 0.5V_{bat,i}^2 & \sum_{i=1}^{N} Q_{i} + I_{i} & \sum_{i=1}^{N} abs(I_{i})exp(t_{i}) \\
\sum_{i=1}^{N} 0.5V_{bat,i} & \sum_{i=1}^{N} 0.5V_{bat,i} & \sum_{i=1}^{N} Q_{i} + I_{i} & \sum_{i=1}^{N} abs(I_{i})exp(t_{i}) \\
\sum_{i=1}^{N} (Q - I_{i}) & \sum_{i=1}^{N} (Q - I_{i}) & \sum_{i=1}^{N} Q_{i} + I_{i} & \sum_{i=1}^{N} abs(I_{i})exp(t_{i}) \\
\sum_{i=1}^{N} abs(I_{i})exp(t_{i}) & \sum_{i=1}^{N} abs(I_{i})exp(t_{i}) & \sum_{i=1}^{N} abs(I_{i})exp(t_{i}) & \sum_{i=1}^{N} (abs(I_{i})exp(t_{i}))^2
\end{bmatrix}
$$

(12)

Therefore, the optimum (battery discharge) coefficients given by the $x$ vector can be determined for each of the four segments that make up the actual discharge curve of the battery in lab conditions, as shown in Figure 6.

This task is accomplished by the selection of a proper window comprising a finite number of the lab measurements for each of the four segments such that $8 < N < 15$ data points and then running recursively the filter in that $N$ sized sample data base.

Initially, the window sizing of the filter shall be selected as $N=8$, which shall produce the solution of compromise of maximum filter sensitivity with close tracking of the actual discharge curve, provided the modeling of the resistance $R$ is indeed that of a second order polynomial.

Once that a given batch of $N$ discharge points is processed, and the optimal coefficients $x_{opt}$ are determined, then it is time to propagate the filter, such that another new datum of the discharge curve, i.e. $V_{bat N+1}$ is to be processed.
Therefore, the window size \( N \) shall become equal to 9 and one cannot, as a rule of thumb, be sure that the optimum coefficients just determined above shall be one and the same for the previous batch.

So, the optimization problem can now be restated under a slightly different enunciate as,

Minimize,

\[
I_p^* = \sum \left( V_{\text{batt}} - E_0 - (R_{\text{opt}} + \Delta R) I_i - (K_{\text{opt}} + \Delta K) \times \frac{Q(I_{It} + I_i)}{[Q - I_{It}]} \right) + (B + \Delta R) \times |I_i| \times \exp(t_i)^2
\]

The necessary conditions for the existence of the minimum and optimum set of model parameters \( \{\Delta a_0, \Delta a_1, \Delta a_2, \Delta K, \Delta B\} \) are likewise given by,

\[
\frac{\partial I_p}{\partial a_0} = 0; \frac{\partial I_p}{\partial a_1} = 0; \frac{\partial I_p}{\partial a_2} = 0; \frac{\partial I_p}{\partial K} = 0; \frac{\partial I_p}{\partial B} = 0
\]

These equations can, now, be individually expressed as,

\[
\frac{\partial I_p}{\partial a_0} = \sum \left[ V_{\text{batt}} - E_0 - (R_{\text{opt}} + \Delta R) I_i - (K_{\text{opt}} + \Delta K) \times \frac{Q(I_{It} + I_i)}{[Q - I_{It}]} \right] (B + \Delta R) \times |I_i| \times \exp(t_i) \times 1 = 0
\]

\[
\frac{\partial I_p}{\partial a_1} = \sum \left[ V_{\text{batt}} - E_0 - (R_{\text{opt}} + \Delta R) I_i - (K_{\text{opt}} + \Delta K) \times \frac{Q(I_{It} + I_i)}{[Q - I_{It}]} + (B_{\text{out}} + \Delta R) \times |I_i| \times \exp(t_i) \right] \times V_{\text{batt}} = 0
\]

\[
\frac{\partial I_p}{\partial a_2} = \sum \left[ V_{\text{batt}} - E_0 - (R_{\text{opt}} + \Delta R) I_i - (K_{\text{opt}} + \Delta K) \times \frac{Q(I_{It} + I_i)}{[Q - I_{It}]} \right] (B + \Delta R) \times |I_i| \times \exp(t_i) \times 0.5V_{\text{batt}}^2 = 0
\]

\[
\frac{\partial I_p}{\partial K} = \sum \left[ V_{\text{batt}} - E_0 - (R_{\text{opt}} + \Delta R) I_i - (K_{\text{opt}} + \Delta K) \times \frac{Q(I_{It} + I_i)}{[Q - I_{It}]} \right] + (B + \Delta R) \times |I_i| \times \exp(t_i) \times Q(I_{It} + I_i) \times |I_i| \times \exp(t_i) = 0
\]

\[
\frac{\partial I_p}{\partial B} = \sum \left[ V_{\text{batt}} - E_0 - (R_{\text{opt}} + \Delta R) I_i - (K_{\text{opt}} + \Delta K) \times \frac{Q(I_{It} + I_i)}{[Q - I_{It}]} \right] (B + \Delta R) \times |I_i| \times \exp(t_i) \times |I_i| \times \exp(t_i) = 0
\]

Once again, the above equations can be expressed under a linear matrix format such that \( A\Delta x = y \), where the \( A \) matrix is the same as previously determined and:
$$\Delta x = \begin{bmatrix} \Delta a_0 \\ \Delta a_1 \\ \Delta a_2 \\ \Delta K \\ \Delta B \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{N} [v_{bat,i} - E_0 - R_{opt} i_i - k_{opt} Q(l_i + i_l) + B_{opt} + \text{abs}(l_i) \exp(t_i)] \\ \sum_{i=1}^{N} [v_{bat,i} - E_0 - R_{opt} i_i - k_{opt} Q(l_i + i_l) + B_{opt} + \text{abs}(l_i) \exp(t_i)] \text{abs}(l_i) \exp(t_i) \end{bmatrix}$$

Now, the corrections computed by $\Delta x$ can be incorporated to $x_{opt}$ then generating $x_{opt, new} = x_{opt} + \Delta x$.

In case that a new interaction is sought for more precision, then the above algorithm can be reiterated and yet another $\Delta x$ correction can be determined and added to $x_{opt, new}$ and so on.

The convergence criterion for accomplishing another interaction – or not = is such that $\Delta R$ is in the range $2.5\% < \Delta R < 5\%$. Another way to control this numerical precision of the $R_{opt}$ values is to let the window size to go increasing until $N=15$ samples, such that the numerical filter estimates improve their numerical stability, in spite of the slight bigger computer overhead.

The $2.5\% < \Delta R < 5\%$. range was chosen by the author for both Physics and Engineering good practices.

So, whenever the numerical filter converges in that range, the algorithm shall increase one more lab measurement data point, when ingesting the $N+2$th data point –and so on - until $N=15$ is reached, thus allowing that the filter shall run on its maximum precision, until preferably all lab data samples are processed under that $N=15$ sliding window.

However, in the “knees” - or breaks - of the cloud of the lab data samples, such as in Fig 6, the filter will allow a large overshoot, perhaps starting to divert significantly from that cloud. In this case, for each newly ingested lab measurement data, the algorithm shall then discard the two oldest processed points such that the order $N$ of the window is decreased by one unit such that filter precision is continuously traded by accuracy.

Such process shall proceed in the neighborhood of the knees of the data cloud until the correct data tracking is restored. The minimum $N=8$ shall be enough to restore such behavior.

Finally, once that the tracking is restored, then the algorithm shall start to accumulate the newer data points to be processed, thus increasing the window size $N$ again until $N=15$ for maximum precision is restored. And so on, until the next knee of the data cloud is reached, etc.

VI. CONCLUSION AND FUTURE WORK

This work resulted in a complete development of an intelligent battery charger, based in Half-Bridge converter, for a semi-autonomous mobile robot along with a study showing the effectiveness of the SOC algorithms. An experiment was made to validate the effectiveness of the battery model used in SimPowerSystems described by the ref. [7]. The analysis showed that the internal resistance cannot be
assumed to be constant in order to have an accurate description of the lead acid battery. Then, this same model was extended including the effect of the variable battery resistance using the model of Ref [14]. After an analysis the new model showed promising results and more accurate SOC estimation.

A future work in this theme could be developed by studying the charge voltage and current curves of the battery and commenting the results by means of a Linear Quadratic estimator as also presented here such Linear Quadratic estimator shall be used to determine the optimal parameters of the model proposed by Ref [14] to fit the observed lab values of the battery discharge.

An wholly analogous Linear Quadratic filter can also be developed for the battery charging model proposed by Ref[14]

Also, in order to increase even further the precision of the models an Extended Kalman Filter and a Particle Filter, as presented by ref [2], could be applied in order to estimate the SOC considering the Peukert Effect and the battery temperature.

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REFERENCES


